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Resonant activation in the presence of nonequilibrated baths

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We study the generic problem of the escape of a classical particle over a fluctuating barrier under the influence of non-Gaussian noise mimicking the effects of nonequilibrated bath. The model system is described by a Langevin equation with two independent noise sources, one of which stands for the dichotomous process and the other describes external driving by α-stable noise. Our attention focuses on the effect of the structure of stable noises on the mean escape time and on the phenomenon of resonant activation. Possible physical interpretation of the occurrence of Lévy noises and the relevance of the model for chemical kinetics is briefly discussed.

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I. INTRODUCTION

In a classical Langevin approach the influence of the bath of surrounding molecules on a Brownian particle is described in terms of a mean-field, time dependent stochastic force which is commonly assumed to be white Gaussian noise. That postulate is compatible with the assumption of a short correlation time of fluctuations, much shorter than the time-scale of the macroscopic motion and assumes that weak interactions with the bath lead to independent random variations of the parameter describing the motion. In more formal, mathematical terms Gaussianity of the state-variable fluctuations is a consequence of the central limit theorem which states that normalized sum of independent and identically distributed random variables with finite variance converges to the Gaussian probability distribution. If, however, after random collisions jump lengths are ruled by broad distributions leading to the divergence of the second moment, the statistics of the process changes significantly. The existence of the limiting distribution is then guaranteed by the generalized Lévy-Gnedenko [1] limit theorem. According to the latter, normalized sums of independent, identically distributed random variables with infinite variance converge in distribution to the Lévy statistics. At the level of the Langevin equation, Lévy noises are generalization of the Brownian motion and describe results of strong collisions between the test particle and the surrounding environment. In this sense, they lead to different models of the bath that go beyond a standard "close-to-equilibrium" Gaussian description.

As documented elsewhere [2,3], not fully thermalized systems or systems driven away from the equilibrium can manifest interesting physical properties. In particular, such systems may exhibit large energy fluctuations with probabilities higher than those predicted by the Gaussian statistics. Nonequilibrated heat reservoir can be thus considered as a source of non-Gaussian noises. Formalisms that give physical background for this phenomena are based on the idea of nonextensive thermodynamics, established on a Tsallis statistics [4], and the fractional Fokker-Planck equation (FFPE) [5]. The latter (FFPE) have been successfully applied [5–9] for describing anomalous diffusion processes, for which the non-local character brought about by the stable Lévy noise leads to the replacement of the local spatial derivatives in the diffusion term of the Fokker-Planck equation by a fractional derivative.

In contrast to the fractional calculus, the Tsallis statistics approach to stable, Lévy-type fluctuations is based on the introduction of a special entropy form [4] from which examined thermodynamical quantities can be derived. The main property of the Tsallis entropy is its nonextensivity, i.e., the entropy of the system containing two noninteracting subsystems is different from the sum of the subsystems' entropies typical for equilibrium Gibssian ensembles and Gaussian measures.

Without discussing the origin of non-Gaussian driving force and its relation to the nonequilibrium thermodynamics in a full extent, we rather focus here on the approach based on the generalized Langevin equation that incorporates α-stable noises as additive forces acting on the system of interest. In fact, various examples of everyday life phenomena and a wide extent of experimental observations [2,10,11] show existence of long-range correlations, disorder, cooperativity, and deviations from the Gaussian statistics, thus suggesting a strong need to study more general probability distributions than just Gaussian ones. In particular, α-stable distributions have been observed in anomalous dynamics and strange kinetics in amorphous semiconductors and glassy systems [8]. Lévy-flight models turn out to be adequate for the description of transport in heterogeneous catalysis, self-diffusion in micelle systems and analysis of geophysical data [12–14]. Related models have been also applied in financial modeling and analysis of economic time series [15]. Among various aspects of the Lévy-type variables and processes, a problem of special interest is numerical generation of stable distributions and simulation of α-stable integrals and stochastic differential equations [15–18]. As broadly discussed in literature [16,18], crucial difficulties in solving stochastic differential equations with stable measures are caused by the noisy term, which allows for larger fluctuations with higher probabilities than Gaussian distributions. Numerical methods [16] for such equations are more sophisticated than for differential equations [19] and for stochastic differential equations with Gaussian noises [20]. In particular, the nonexistence of variance for stable variables makes the problem

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much more complicated to tackle both, numerically [16] and analytically [21,22]. It turns out, however, that with the use of suitable statistical estimation techniques, computer simulation procedures, and numerical discretization methods, it is possible to construct relevant approximations of stochastic integrals with stable measures as integrators [16,17].

In this paper we study statistical properties of the generic system describing passage of a classical particle over a fluctuating potential barrier. The system is coupled to a non-Gaussian bath modeled by the Lévy stable noise. We present numerical results for the mean first passage time of the particle over the barrier, assuming a linear potential subject to Markovian dichotomous fluctuations. Our model belongs to the class of "on-off" models discussed in a paper by Doering and Gadoua [23] and further analyzed by Boguñá *et al.* [24]. A distinctive characteristics of this model is that part of the time the barrier is either switched off (i.e., it becomes flat) or the switching is performed between the barrier and a well, so that the particle can essentially roll rather than climb during these times. The main difference between the model considered here is a form of driving, additive fluctuations. Whereas in the previous papers mainly white Gaussian noises have been considered [23,25–28], here additive noises are assumed to be α-stable [17], which might arise from the contact with nonequilibrated bath.

The problem of resonant activation (RA) [23] examined in this paper is an example of processes manifesting constructive role of noise [29]. The paper deals with a modified version of the model proposed by Doering and Gadoua [23]. Sec. II presents the model and poses the problem to be studied. In Secs. III and IV general considerations of Lévy noises in physical systems are addressed and the problem of underlying thermodynamic interpretation is briefly discussed. Results of simulations with a short note on numerical methods applied for generating stable variables and integration of stochastic differential equations with stable measures are included in Sec. V. The paper is closed with the concluding remarks.

II. GENERIC MODEL SYSTEM

We consider an overdamped Brownian particle moving in a potential field between absorbing ($x=1$) and reflecting ($x=0$) boundaries (cf. Fig. 1), in the presence of noise that modulates the barrier height.

Time evolution of a state variable $x(t)$ is described in terms of the Langevin equation

$$\frac{dx}{dt}=-V'(x)+g\,\eta(t)+\zeta(t)=-V'_{\pm}(x)+\zeta(t), \qquad (1)$$

where prime means differentiation over x, $\zeta(t)$ is a white Lévy process originating from the contact with nonequilibrated bath, and $\eta(t)$ stands for a Markovian dichotomous noise of intensity g taking one of two possible values ± 1. Autocorrelation of the dichotomous noise is set to $\langle[\eta(t)-\langle\eta\rangle][\eta(t')-\langle\eta\rangle]\rangle=\exp(-2\gamma|t-t'|)$. For simplicity, throughout the paper a particle mass, a friction coefficient, and the Boltzmann constant are all set to 1. The time-dependent potential $V_{\pm}(x)$ is assumed to be linear with the barrier switching between two configurations with an average rate γ (cf. Fig. 1),

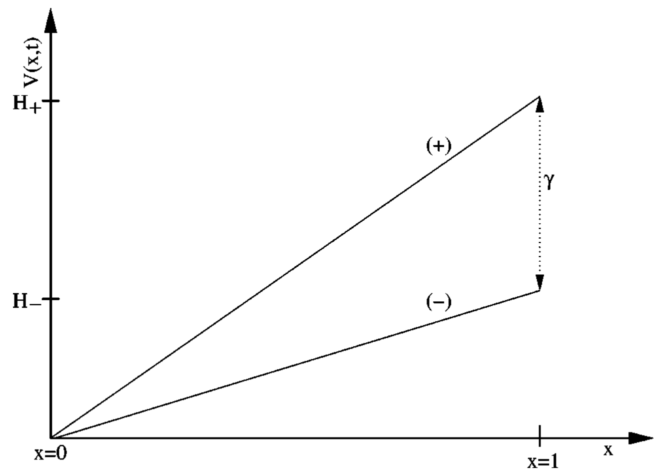

FIG. 1. A model potential studied in the paper. The barrier height fluctuates dichotomously between the values $H_{\pm}$. A particle starts its diffusive motion at a reflecting boundary $x=0$ and continues until the absorption at $x=1$.

$$V_{\pm}(x)=H_{\pm}x, \quad g=\frac{H_{-}-H_{+}}{2}, \qquad (2)$$

both ζ and η noises are assumed to be statistically independent.

The initial condition for Eq. (1) is

$$x(0)=0, \qquad (3)$$

i.e., initially particle is located at the reflecting boundary with equal choices of finding a potential barrier in $(\pm)$ configurations

$$P(H_{+},t=0)=P(H_{-},t=0)=\tfrac{1}{2}. \qquad (4)$$

The quantity of interest is the mean first passage time (MFPT), τ

$$\tau=\int_0^1 dx\int_0^\infty [p_{-}(x,t)+p_{+}(x,t)]dt, \qquad (5)$$

i.e., the average time which particle spends in the system before it becomes absorbed. Within the proposed approach, the MFPT is estimated as a first moment of the distribution of first passage times (FPT) obtained from the ensemble of simulated realizations of the stochastic process in question. Otherwise, for Lévy flights described by Eq. (1), the MFPT may be calculated after solving a relevant deterministic FFPE [30] for the distribution function

$$\frac{\partial p_{\pm}(x,t)}{\partial t}=\frac{\partial}{\partial x}\left[\frac{\partial V_{\pm}(x)}{\partial x}p_{\pm}(x,t)\right]+D\nabla^{\alpha}p_{\pm}(x,t)+\gamma p_{\mp}(x,t)-\gamma p_{\pm}(x,t). \qquad (6)$$

In the above FFPE $p_{\pm}(x,t)$ are probability density functions (PDFs) for finding a particle at time t in the vicinity of x, while potential takes the value $V_{\pm}(x)$. The fractional derivatives ∇^{α} is understood in the sense of the Fourier transform [31]

$$\nabla^{\alpha} = -\int \frac{dk}{2\pi} e^{ikx} |k|^{\alpha}, \tag{7}$$

with $\alpha = 2$ corresponding to the standard Brownian diffusion. The coefficient D denotes the generalized diffusion coefficient with the dimension [30] $[D] = \mathrm{cm}^{\alpha}\ \mathrm{sec}^{-1}$ and can be related to the parameter σ characterizing the width of the PDF (see below). In the approach presented herein, instead of solving equation (6), information on the MFPT is drawn from the statistics of numerically generated trajectories satisfying the Langevin equation (1). Before proceeding further, we remind shortly basic definitions and formalisms related to the α-stable statistics and noises.

III. LÉVY-TYPE VARIABLES

The α-stable variables are random variables for which the sum of random variables is distributed according to the same distribution as each variable, i.e.,

$$aX_1 + bX_2 \overset{\mathrm{d}}{=} cX + d, \tag{8}$$

where $\overset{\mathrm{d}}{=}$ denotes equality in a distribution sense. Real constants c, d in Eq. (8) allow for rescaling and shifting of the initial probability distribution.

The characteristic function of the probability distribution that fulfills Eq. (8) can be parametrized in various ways. In the usually chosen $L_{\alpha,\beta}(\zeta;\sigma,\mu)$ [17,18] parametrization, a characteristic function of the Lévy-type variables is given by

$$\phi(k) = \exp\left[-\sigma^{\alpha}|k|^{\alpha}\left(1 - i\beta\ \mathrm{sgn}(k)\tan\frac{\pi\alpha}{2}\right) + i\mu k - i\beta k\sigma^{\alpha}\tan\frac{\pi\alpha}{2}\right] \quad \text{for} \quad \alpha \neq 1,$$

$$\phi(k) = \exp\left[-\sigma|k|\left(1 + i\beta\frac{2}{\pi}\ \mathrm{sgn}(k)\ln|k|\right) + i\mu k\right], \quad \text{for} \quad \alpha = 1, \tag{9}$$

with $\alpha \in (0,2]$, $\beta \in [-1,1]$, $\sigma \in (0,\infty)$, $\mu \in (-\infty,\infty)$, and $\phi(k)$ defined in Fourier space

$$\phi(k) = \int d\zeta e^{-ik\zeta} L_{\alpha,\beta}(\zeta;\sigma,\mu). \tag{10}$$

Parameter α is called the stability index, β describes skewness of the distribution, σ is responsible for its scaling and μ is a location parameter. The above parametrization (9) is continuous, in the sense that

$$\lim_{\alpha\to 1} \sigma^{\alpha}[|k|^{\alpha}\ \mathrm{sgn}(k) - k]\tan\frac{\pi\alpha}{2} = -\frac{2}{\pi}\sigma|k|\mathrm{sgn}(k)\ln|k| \tag{11}$$

for every σ and k.

Although the PDFs for stable variables $L(\zeta)$ are known to have an asymptotic power-law behavior $L(\zeta) \sim |\zeta|^{-(\alpha+1)}$, analytical expressions corresponding to Eq. (9) can be given only in few cases. In particular, for $\alpha = 2, \beta = 0$, ζ is a Gaussian variable with the probability density

$$L_{2,0}(x;\sigma,\mu) = \frac{1}{2\sigma\sqrt{\pi}} \exp\left(-\frac{(x-\mu)^2}{4\sigma^2}\right), \tag{12}$$

whereas $\alpha = 1, \beta = 0$, and $\alpha = \frac{1}{2}$, $\beta = 1$ yield Cauchy

$$L_{1,0}(x;\sigma,\mu) = \frac{\sigma}{\pi} \frac{1}{(x-\mu)^2 + \sigma^2} \tag{13}$$

and Lévy-Smirnoff $(x > \mu)$

$$L_{1/2,1}(x;\sigma,\mu) = \left(\frac{\sigma}{2\pi}\right)^{1/2} (x-\mu)^{-3/2} \exp\left(-\frac{\sigma}{2(x-\mu)}\right) \tag{14}$$

distributions, respectively.

Generally, for $\beta = \mu = 0$ PDFs are symmetric and for $\beta = \pm 1$ and $\alpha \in (0,1)$ they are totally skewed [17].

IV. PHYSICAL INTERPRETATION

By definition, in the case of static barrier height ($\eta = 0$), with the noise ζ uncorrelated at different times and obeying the Lévy statistics, the overdamped Langevin equation (1) describes Lévy flights [30,31] in a constant force field H_0. For long times, the trajectory $x(t)$ behaves as

$$x(t) \approx H_0 t + \int_0^t ds\,\zeta(s) \tag{15}$$

and yields the Lévy stable distribution [31] in the position of the particle. In consequence, if the first moment exists, i.e., for $1 < \alpha \leqslant 2$, mean value of $x(t)$ grows linearly with time, $\langle x(t)\rangle = H_0 t$, whereas the mean-square displacement becomes $\langle [x(t) - \langle x(t)\rangle]^2\rangle = 2D = 2\sigma^2$ only for $\alpha = 2$ when the finite second moment of $L(\zeta)$ exists. Thus the generalized Einstein relation connecting the first moment in the presence of constant force H_0 to the second moment in the absence of force $\langle x(t)\rangle_{H_0} = H_0\langle x^2(t)\rangle_0$ is recovered only in the Brownian limit $\alpha = 2$ with the noise amplitude σ related to the diffusion coefficient $\sigma = D^{1/2}$. Obviously, for ζ noises with diverging mean-square displacement, the classical fluctuation dissipation theorem is violated, and the Einstein relation does not hold longer [9,31,33].

When analyzed from the perspective of the continuous time random walks (CTRW), Lévy flights characterize walks with a Poisson waiting time and a Lévy distribution of the jump length. The scaling nature of the jump length PDF leads then to a clustering of the Lévy flights visible via interruption of the local motion by occasional long sojourns on all length scales [5]. This fractal character of the Lévy flight trajectory can be contrasted with subdiffusive CTRW [5,9] that fills the two-dimensional space completely and features no clusters. In such a case, however, the time intervals between consecutive steps are governed by the power-law waiting time distributions that lead to a sublinear dependence $\langle x(t)\rangle \approx t^{\nu}$ with ν denoting the power-law index of the waiting time PDF. Therefore, subdiffusive CTRW in a constant external force acting along the x direction perfectly satisfies the fluctuation-dissipation theorem [5,9] which holds also in the corresponding fractional Fokker-Planck equation framework.

Diverging mean-square displacement cannot be valid for a particle with nondiverging mass. In fact, for massive particles, a finite velocity of propagation exists making very long instantaneous jumps impossible. For that reason, the dilemma of diverging mean-square displacement in the Lévy flight can be overcome by CTRW version of Lévy walks with a suitable time cost penalizing long jumps. Nevertheless, in many physical systems of interest diffusion of state variable $x(t)$ with diverging second moment does not violate physical principles and is a legitimate way of physical modeling [12,13,34]. Therefore, not advocating further use of particular approach to the Lévy-flight models in external fields, we stick here to the direct integration of the generalized Langevin equation (1) with non-Gaussian fluctuating forces.

V. STOCHASTIC DIFFERENTIAL EQUATIONS WITH STABLE NOISES

A stochastic process with independent increments distributed according to the α-stable distribution is known as the standard Lévy motion. As a consequence of the characteristic distribution of increments, such a process is $1/\alpha$ self-similar. In order to investigate statistical properties of a motion described by Eq. (1), the model Langevin equation has been simulated by use of the appropriate numerical methods. Position of the particle is then obtained by direct integration of Eq. (1),

$$x(t) = -\int_{t_0}^{t}[V'(x(s)) - g\,\eta(s)]ds + \int_{t_0}^{t} dL_{\alpha,\beta}(s) = -\int_{t_0}^{t} V'_{\pm}(x(s))ds + \int_{t_0}^{t} dL_{\alpha,\beta}(s). \tag{16}$$

In general [17,22], the $L_{\alpha,\beta}$ measure in Eq. (16) can be approximated by

$$\int_{t_0}^{t} f(s)dL_{\alpha,\beta}(s) \approx \sum_{i=0}^{N-1} f(i\Delta s)M_{\alpha,\beta}([i\Delta s,(i+1)\Delta s)) \stackrel{\mathrm{d}}{=} \sum_{i=0}^{N-1} f(i\Delta s)\Delta s^{1/\alpha}\varsigma_i, \tag{17}$$

where ς_i is distributed with the PDF $L_{\alpha,\beta}(\varsigma;\sigma,\mu=0)$, $N\Delta s = t - t_0$, and $M_{\alpha,\beta}([i\Delta s,(i+1)\Delta s))$ is the measure of the interval $[i\Delta s,(i+1)\Delta s)$.

Random variables ς corresponding to the characteristic function (9) can be generated using the Janicki-Weron algorithm [16,18]. For $\alpha \neq 1$ their representation is

$$\varsigma = D_{\alpha,\beta,\sigma}\frac{\sin[\alpha(V + C_{\alpha,\beta})]}{[\cos(V)]^{1/\alpha}} \times \left[\frac{\cos[V - \alpha(V + C_{\alpha,\beta})]}{W}\right]^{(1-\alpha)/\alpha} + B_{\alpha,\beta,\sigma,\mu}, \tag{18}$$

with constants B,C,D given by

$$B_{\alpha,\beta,\sigma,\mu} = \mu - \beta\sigma^{\alpha}\tan\left(\frac{\pi\alpha}{2}\right), \tag{19}$$

$$C_{\alpha,\beta} = \frac{\arctan\left[\beta\tan\left(\frac{\pi\alpha}{2}\right)\right]}{\alpha}, \tag{20}$$

$$D_{\alpha,\beta,\sigma} = \sigma\left\{\cos\left[\arctan\left[\beta\tan\left(\frac{\pi\alpha}{2}\right)\right]\right]\right\}^{-1/\alpha}. \tag{21}$$

For $\alpha = 1$, ς can be obtained from the formula

$$\varsigma = \frac{2\sigma}{\pi}\left[\left(\frac{\pi}{2} + \beta V\right)\tan(V) - \beta\ln\left(\frac{\frac{\pi}{2}W\cos(V)}{\frac{\pi}{2} + \beta V}\right)\right] + B_{1,\beta,\sigma,\mu}, \tag{22}$$

with

$$B_{1,\beta,\sigma,\mu} = \mu + \frac{2}{\pi}\beta\sigma\ln(\sigma). \tag{23}$$

In the above equations V and W are independent random variables, such that V is uniformly distributed in the interval $(-\pi/2,\pi/2)$ and W is exponentially distributed with a unit mean [17,18].

The problem described by Eq. (1) and corresponding Eq. (16) for the stability index $\alpha = 2$ is a well known case of resonant activation resolved in the series of papers [23,27,28]. Accordingly, for $\alpha = 2$ integration in Eq. (16) is performed with respect to a Gaussian measure and standard numerical methods can be applied to solve the Langevin equation under study. Otherwise, the ordinary forward (or backward) in time Fokker-Planck equation for the PDF can be derived in a closed form, from which the MFPT can be easily calculated. Also, for the Gaussian fluctuations theoretical background is provided by the standard thermodynamics where parameter σ can be related to the intensity of the thermal fluctuations imposed on a physical mode x.

Within these studies, besides the driving Gaussian fluctuations, two other kinds of α-stable noises have been considered, namely, the Cauchy noise ($\alpha = 1$, $\beta = 0$) and the Smirnoff noise ($\alpha = 0.5$, $\beta = 1$) with intensities $\sigma = 0.5$, $\sigma = 1/\sqrt{2}$, and $\sigma = 1$. The value of the location parameter μ has been arbitrary set to 0.

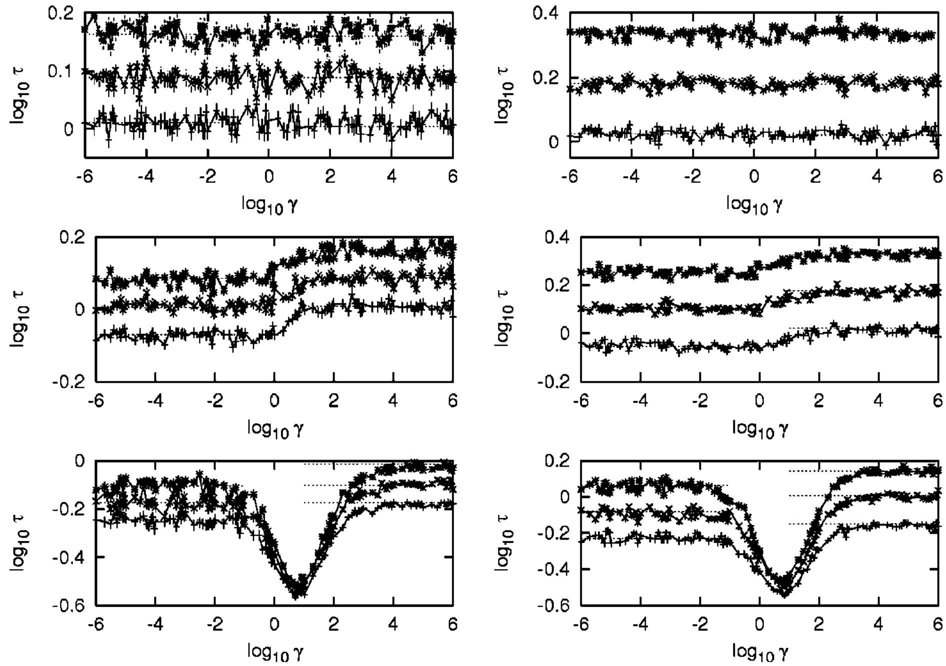

FIG. 2. The $\tau(\gamma)$ as a function of barrier fluctuation rate γ for linear potentials with $H_+=8$, $H_-=0$ (middle panel); $H_+=8$, $H_-=4$ (upper panel); $H_+=8$, $H_-=-8$ (lower panel) are plotted along with asymptotic lines. The additive noise is of the Lévy-Smirnoff type (left panel) ($\alpha=0.5$, $\beta=1$, $\mu=0$) and of the Cauchy type (right panel) ($\alpha=1$, $\beta=0$, $\mu=0$), respectively. Different symbols correspond to varying intensity of fluctuations: $\sigma=0.5$ (+), $\sigma=1/\sqrt{2}$ ($\times$), and $\sigma=1$ (*). Numerical results have been obtained by use of Eq. (16) with the time step $dt=10^{-5}$. Noises were generated according to Eqs. (18) and (22). In order to gain sufficient statistics averaging of Eq. (16) over 10^3 realizations has been performed. Asymptotic lines have been evaluated for reference static potential profiles with the time step $dt=10^{-5}$ and averaged over 10^4 realizations. Error bars have been estimated by use of standard techniques for the Monte Carlo data analysis [36]. They represent standard deviation from the mean. Lines have been drawn to guide the eye.

The choice of σ, which scales the distribution width and varies the intensity of fluctuations, corresponds, although not in a self-transparent way, to the change of system temperature. However, the problem of definition of the system temperature is more subtle here than in the presence of a standard bath [4,32,35] because the system is not in the state of equilibrium.

Lévy-type variables have been generated by use of recipes given by Eqs. (18) and (22). The trajectory $x(t)$ has been generated according to Eq. (16) for various noise realizations. Its evaluation proceeded till the time t' when $x(t')\geqslant 1$ for the first time. The resulting distribution of FPT has been further used to evaluate MFPT.

The linear potential $V_\pm=H_\pm x$ has been dichotomously alternating between different values of $H_\pm$. Our simulations have been obtained for $H_\pm=\pm 8$ [cf. Fig. 2 (lower panel)], $H_-=0$, $H_+=8$ [cf. Fig. 2 (middle panel)] and $H_-=4$, $H_+=8$ [cf. Fig. 2 (upper panel)].

Asymptotic lines plotted in Fig. 2 have been calculated numerically by use of the Monte Carlo method [36]. As expected, typical behavior [23,24,27] has been recovered for $\gamma\to 0$

$$\tau(\gamma\to 0)=\frac{1}{2}[\tau(H_-)+\tau(H_+)], \tag{24}$$

and for $\gamma\to\infty$

$$\tau(\gamma\to\infty)=\tau\left(\frac{1}{2}(H_-+H_+)\right). \tag{25}$$

Figure 2 presents results of simulations averaged over 10^3 realizations with a time step $\Delta t=10^{-5}$. For calculation of the asymptotic lines [Eqs. (24) and (25)] $\Delta t=10^{-5}$ and averaging over 10^4 realizations has been performed.

For the barrier switching between $H_\pm=\pm 8$, the phenomenon of resonant activation is clearly visible cf. Fig. 2 (lower panel). By comparison to the results obtained for a motion of a particle subject to the additive white Gaussian noise [23–27], the value of the MFPT is smaller and resonant activation is observed at lower frequencies γ. Moreover, for $H_\pm=\pm 8$, asymptotic values of MFPT estimated for $\gamma\to\infty$ and $\gamma\to 0$ are higher (lower) than in the corresponding white Gaussian-noise case [23,24,27].

For other cases under consideration, i.e., for $H_-=0$, $H_+=8$ and $H_-=4$, $H_+=8$, the resonant activation has not been observed [cf. Fig. 2 (middle and upper panel)]. However, values of reported MFPTs for the system driven by non-Gaussian Lévy noises are always significantly smaller than in the case of Gaussian-noise driving. It is caused by the fact that the Cauchy and Lévy-Smirnoff noises allow for higher values of driving fluctuations in the Langevin equation (1).

As it can be inferred from Fig. 2, values of the MFPT for the Cauchy noise are higher than the corresponding times for the Lévy-Smirnoff noise, compare the left and the right panel of Fig. 2. This observation can be explained by the difference between both statistics: for the given sets of parameters, the Lévy-Smirnoff distribution is more "heavily tailed" than the Cauchy distribution.

The procedure applied for numerical integration of Eq. (16) is valid for every allowed value of α. In particular, the case with $\alpha=2$ has been investigated along these lines in order to test the numerical approach adopted in simulation of Eq. (1). The test concluded a perfect agreement of numerically simulated results with formerly solved Gaussian cases of RA [23,26–28].

VI. SUMMARY

We have considered a thermally activated process that occurs in a system coupled to a non-Gaussian noise source introduced by a nonequilibrated thermal bath. Another external stochastic process is assumed to be responsible for dichotomous fluctuations of the potential barrier which has been modeled by the linear function with a varying slope.

In comparison to the Gaussian case [28], when the RA phenomenon has been observed for all barrier setups ($H_{\pm}=\pm 8$, $H_{+}=8$, $H_{-}=0$, $H_{+}=8$, $H_{-}=4$) analyzed in this study, non-Gaussian additive stable noises produce resonant activation observable only for the $H_{\pm}=\pm 8$ case. Resulting MFPTs are significantly smaller than those obtained for the Gaussian source of fluctuations. The effect is due to higher probabilities of the extreme events (large fluctuations) allowed by the Lévy statistics. Similarly to the Gaussian-bath case, a typical asymptotic behavior of the MFPT has been recovered for large and small frequencies of the dichotomous noise: for small γ MFPT tends to the average MFPTs for the both barrier configurations, while for large γ it becomes equal to the MFPT over the average potential barrier [23,25,26].

These observations have several implications in relation to chemical kinetics in conformationally varying media [38–40] where flipping barriers separating reactants' and products' basins may be due to a dynamic isomerization of the activated complex. The effects of fluctuations in force or potential on the diffusive process have been also extensively studied in the context of motor proteins [41]. Under typical conditions, i.e., with a Gaussian additive thermal noise and with a flipping barrier height, the RA phenomenon is registered in all those realms as a maximal flux of particles or a maximum reaction rate. In contrary to this finding, the analogous systems influenced by the presence of non-Gaussian additive noises exhibit RA only for potentials switching between the barrier and the well (Fig. 2, lower panel). Otherwise, the kinetics seems to be fairly insensitive to the flipping rate of an erecting barrier (Fig. 2, middle panel) or becomes slowed down (Fig. 2, upper panel) when the frequency of switching between a high and a low barrier becomes large. We should mention here that so far, our numerical results concerned only special choices of α, β parameters that define the α-stable distributions. Therefore, it cannot be excluded that for other (α,β) sets in the α, β space, the RA phenomenon would be observable for all barrier setups. In fact, preliminary studies [42] suggest that the phenomenon can be reinstalled for fully asymmetric noises with low α values.

In addition, physical interpretation of the Lévy noisy term in Eq. (1) is brought sometimes with a relevance to the nonextensive thermodynamics and the Tsallis statistics [4,5,30,31,35]. In general, however, descriptions provided by the FFPE approach and the Tsallis statistics are not in a full agreement [5,30,31]. Nevertheless, the formalism of nonextensive statistical mechanics (or the "superstatistics" concept, in general, Ref. [35]) offers an intriguing interpretation of non-Gibssian ensembles that can model nonequilibrated baths. Especially, for nonquilibrium systems composed of regions that exhibit spatiotemporal fluctuations of an intensive quantity (such as pressure, chemical potential, inverse temperature, or the energy dissipation rate [35]) generalized statistics may emerge in consequence of a statistical subordination of the intensive variable to those fluctuations.

The situation in which resulting probability distributions are not Gaussian are frequent in various physical situations. The phenomena of special interest are random walks that may lead to Lévy distributions [5] and hence provide a possible realization of Lévy noise sources.

There are several candidates for such a noise origin to be taken into account. Obviously, all noise sources need to be infinite to allow, with a nonzero probability, infinite fluctuations. Among possible models of chaotic nonequilibrium baths perhaps the most natural is a model of the fluidlike bath [37] abruptly perturbed with a local heating to a very high temperature. Gradual spreading of energy progresses until a new equilibrium state is reached and for some finite periods of time, the probing system that experiences local change of the bath temperature will be governed by non-Boltzmann statistics [32,37]. This and similar approaches validate then use of the Lévy-type statistics for description of systems functioning far from equilibrium or subject to nonequilibrated baths. As known from other similar studies [22], the Lévy-type structure of the noise imposed on a double-well potential can affect profoundly noise induced jumping between metastable states and result in stationary PDF deviating severely from the usual Gibbs distribution.

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